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with Entry and Exit events**

Camilla Ferretti

Quaderno n. 124/aprile 2017

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Abstract

This paper faces the problem of measuring mobility in a set of individuals evolving with the time among k states, when entry/exit phenomena are considered. Movements among the states are usually reckoned through the corresponding transition matrix P and mobility is measured by a given index $I(P)$. We propose here a procedure based on the hypothesis that entering/exiting individuals make the mobility respectively increase/decrease. Such procedure exploits existing mobility indices for measuring the Incumbents mobility, according with their starting state, and weighs the contribute of every state using the information about the corresponding entry/exit rate. The new index is proved to satisfy properties of boundedness, immobility and perfect mobility.

Keywords: Transition matrix, mobility index, entry/exit events.

1 Introduction

In this work we face the problem to measure the mobility in a sample of individuals evolving with respect to time, when birth and death phenomena are admitted in the dynamics under study. In the past literature, many authors focused on the issue of measuring the degree of mobility in evolving samples. Typically, a set of k non-overlapping states is given, based on a economically relevant variable (for example employment or unemployment, firm size in terms of number of workers, income classes), and transitions between t and $t + 1$ are recorded (for example: one quarter, one year). Measuring the degree of mobility corresponds to the choice of a suitable descriptive statistics able to summarize in a unique real number the global amount of movements. There are mainly two distinct ways to face this issue: *t*) by comparing the distribution among states at time t and $t + 1$ and measuring their distance, as in Shorrocks (1982) and Fields and Ok (1996) (among others); *tt*) by applying a suitable function $I : \mathbb{R}^{k \times k} \rightarrow \mathbb{R}$ on the transition matrix $P = \{p_{ij}\}$ as, for example, in Prais (1955); Adelman (1958); Shorrocks (1978); Parker and Rougier (2001); Bourguignon and Morrison (2002); Alcalde-Unzu *et al.* (2006); Ferretti and Ganugi (2013); Paul (2016).

Along the line outlined in a sequence of previous works (Ferretti, 2012; Ferretti and Ganugi, 2013; Ferretti, 2014), we focus here on the mobility measured through transition matrices (TMs). From this point of view, several features of mobility indices have been already analyzed: for example their relationship with theoretical stochastic processes (Shorrocks, 1976; Sommers and Conlinsk, 1979; Geweke *et al.*, 1986), or their sampling properties (Maaoui and Trede, 2001; Schluter, 1998; Formby *et al.*, 2004; Ferretti, 2014).

The novelty of this article is to deal with the mobility measurement when TMs are based on data affected by *entry/exit* (or equivalently *birth/death*) events. In fact, in empirical applications, mobility is usually measured considering the sampling TM P observed on the interval $[t, t + 1]$ and defined by

$$p_{ij} = \frac{\text{nr. of individuals being in } i \text{ at time } t \text{ and in } j \text{ at time } t + 1}{\text{nr. of individuals being in } i \text{ at time } t}. \quad (1)$$

Eq. 1 requires that every individual in the sample is observed at *both* time t and $t + 1$ in one of the k chosen states, that is observed individuals belong to a *closed* dataset.

On the contrary, it often happens that datasets are actually *open* and some

statistical units are recorded only at time t or only at time $t + 1$. That is, together with the *Incumbents*¹, we observe some individuals (herein *Outgoings*) "disappearing" from their starting state and some individuals (*Incomings*) "appearing" in their final state. Usually appearance or disappearance is signaled by an empty cell in the dataset, that is a missing value corresponding to time t or time $t + 1$. Here, we will consider the entry/exit phenomena in a more extended way: given a fixed set of k regular states, an individual is said to be Incoming (resp. Outgoing) if its observed value at time t (resp. $t + 1$) is missing or it does not coincide with any regular state. It is always possible to clean data considering only the subset of Incumbents, but results could be biased (Heckman, 1990). Furthermore, in many economically relevant dynamics the entry/exit events represent a not-negligible event: for example, when we analyze the credit ratings dynamics, in which firms or countries evolve according with their rating (AAA, AA, ...), the Default state can not be discarded (Jafry and Schuermann, 2004). In this light, it is evident the importance to include information about Incumbents together with Incomings and Outgoings in the mobility measurement.

In many cases individuals arrive or leave because data arise from *rotating panels* or because they become *non-respondent* (Stasny, 1988). In the first case movements in and out from the data panel are ruled by specific conventions. Here we choose instead to work with evolving samples in which movements among states and birth/death phenomena belong to the same dynamics, as in the examples mentioned above, and they are not due to external causes such as rotating rules. On the other hand, non-respondents represent an endogenous fact because of which individuals leave the group under study and become not-observable also if their value still coincides with a regular state. In this sense, non-respondents are not Outgoings. In this work we will consider disappearing individuals as real Outgoings (for example defaulting firms). The analysis of non-respondent dynamics will be matter for further research.

Here we propose a procedure which permits to obtain a new mobility index able to catch both movements of Incumbents and the contribution to mobility of Incomings and Outgoings. Such procedure consists in measuring the Incumbents' mobility among a set of k regular states, possibly through

¹ The term *Incumbents* is generally referred to a set of firms already in position in a market (Black *et al.*, 2009). For extension we use the same term to indicate individuals belonging to the sample for the whole considered interval of time.

a given already existing mobility index and in reweighing it with suitable weights to include information about the In/Outgoings mobility. This procedure produces a full-fledged mobility index equipped with properties such as boundedness. Also, the proposed index is built in a not-parametric framework, i.e. it does not require any assumption about a possible theoretical underlying model. The paper is organized as it follows: Sect. 2 introduces the issue of mobility measurement with transition matrices and the decomposition of mobility according with the starting state; Sect. 3 proposes a way to reorganize transition matrices when open panels are considered, which will be the basis for the new mobility index. In addition, the basic hypothesis underlying the index and its main properties are treated; Sect. 4 illustrates the main properties of the new index; Sect. 5 provides an empirical application to the Italian and UK labor market; the last Section concludes.

2 Mobility measures on TMs

2.1 A review on mobility indices

As outlined before, mobility indices on TMs are functions $I : \mathbb{R}^{k \times k} \rightarrow \mathbb{R}$ able to summarize in a unique value $I(P)$ all the information about movements which are contained in the k^2 elements of P (k is fixed). Such functions are required to satisfy many properties, among whose we recall *monotonicity*. Monotonicity (M) claims that if " P is less mobile than Q ", then $I(P) < I(Q)$ and it defines the main role of I , which consists in revealing the implicit order among all the transition matrices with the same number of states. Actually, as explained in Ferretti and Ganugi (2013), it is generally not possible to establish *a priori* if P is less mobile than Q without measuring an index, except for couples of TMs belonging to a limited subset. The right procedure for ordering TMs consists in choosing the kind of mobility we are interested in (for example, the tendency to leave the current state), calculating an index able to measure such mobility (for example, the trace index) and ordering matrices with respect to the obtained index value. It is evident that, according to this procedure, mobility indices result to be monotone by construction.

Mobility indices are usually required to satisfy: 1) continuity (C); 2) boundedness (B); 3) immobility (I, which requires $I(Id) = 0$ where Id is the $k \times k$ identity matrix) and 4) perfect mobility (PM such that $I(P)$ is maximal for P corresponding to the maximum degree of mobility, which usually can

be identified a priori²).

In literature there exist many proposals for mobility indices on TMs, some of which are summarized in the following list:

- $I^{tr}(P) = \frac{k - \text{tr}P}{k-1}$, where $\text{tr}P = \sum_i p_{ii}$ (Shorrocks, 1978).
- $I^e(P) = \frac{k - \sum_i |\lambda_i|}{k-1}$, where λ_i are eigenvalues of P (Shorrocks, 1978).
- $I^{b1}(P) = \frac{k}{k-1} \sum_i \pi_i (1 - p_{ii})$, where $\pi = (\pi_1, \dots, \pi_k)$ is the equilibrium distribution such that $\pi P = \pi$ (Bartholomew, 1982).
- $I^{b2}(P) = \frac{k}{k-1} \sum_{i,j} p_{ij} |i - j|$ (Bartholomew, 1982).
- Index of predictability $I^p(P) = \frac{k}{k-1} \left(\sum_{i,j} p_{ij}^2 - 1 \right)$ (Parker and Rougier, 2001).
- Up/downward index $I^u(P) = \sum_i d_i \sum_{j>i} p_{ij}$ and $I^d(P) = \sum_i d_i \sum_{j<i} p_{ij}$, where d_i is the fraction of individuals moving from i (Bourguignon and Morrison, 2002).
- The directional index $I^{dir}(P) = \frac{1}{Z} \sum_i \omega_i \sum_j p_{ij} \cdot \text{sign}(j - i) \cdot v(|j - i|)$, where Z is a normalizing constant, $\{\omega_i\}$ are generic weights and $v(\cdot)$ is a non-negative, increasing function (Ferretti and Ganugi, 2013).
- The extension of $I^{b2}(P)$ given by $I^a(P) = \sum_i \omega_i \sum_j p_{ij} |i - j|^a$, $a > 0$ (Paul, 2016).

As outlined before, I^{tr} is a suitable choice when we are measuring the turbulence, whereas for example $I^{b2}(P)$ and $I^a(P)$ can be considered when we are interested in the *intensity* in changing the state, since they give higher weights to larger jumps ($|i - j|$ increases with the distance between i and j). Lastly, I^u , I^d and I^{dir} consider also the prevailing direction in the dynamics under study.

² For example, when we are measuring the tendency to move away from the current state, we can reasonably assume that mobility is maximal when all the diagonal elements p_{ii} are resp. equal to 0. The trace index satisfies this requirement.

2.2 Decomposition of mobility

The list of indices shown in the previous section is not exhaustive, but it contains some examples which are mostly used in literature. Among them, we recognize that the eigenvalue index $I^e(P)$ differs in some manner from the others (as well as the similar indices $I^2(P) = 1 - |\lambda_2|$ and $I^{det}(P) = 1 - |det(P)|$ examined in Shorrocks, 1978 and Sommers and Conlinsk, 1979). As a matter of fact, it can be attributed to the class of indices measuring the speed of convergence towards the steady-state (Ferretti, 2012), and it is based on the assumption that a Markov Chain is ruling the evolution of individuals among the states (see for example Jafry and Schuermann, 2004 and Violi, 2008).

On the contrary, we choose here to work in a non-parametric framework, in the sense that we aim to build a new index which does not require underlying theoretical assumptions. In consequence of that, p_{ij} is actually considered as a conditional frequency (or probability from a frequentist point of view), instead of a conditional theoretical probability. Following the outlined idea in Ferretti and Ganugi, 2013, Sect. 5.2, it is relevant to note that all the remaining indices in the list except $I^e(P)$ ³ can be decomposed as a sum of k terms, each one describing the contribute to the whole mobility due to the individuals starting from the i -th state, $i = 1, \dots, k$. For example the trace index can be written as $I^{tr}(P) = \frac{k}{k-1} \sum_i \frac{1}{k} (1 - p_{ii})$. Formally speaking, a generic not-parametric index $I(P)$ can be rewritten in the following manner:

$$I(P) = C \cdot \sum_i \omega_i \cdot I_{row}(P_i), \quad (2)$$

where C is a normalizing constant (if needed), ω_i are weights to be assigned to every starting state i , such that $0 \leq \omega_i \leq 1$ for every i and $\sum_i \omega_i = 1$, and $I_{row} : \mathbb{R}^k \rightarrow \mathbb{R}$ is a function measuring the mobility associated to every row P_i of the matrix P .

This decomposition mirrors the fact that every individual starting from the i -th state is equipped with a certain degree of mobility according the probability distribution described by P_i . Such feature will be a basis in the following for proposing a new mobility index. Indeed, it is worth noting that, being all the indices linear combinations of $I_{row}(P_1), \dots, I_{row}(P_k)$, a given index

³ Actually, I^{b_l} is based on the presence of an underlying Markov chain, since it includes the equilibrium distribution π . Nevertheless, it is still decomposable and it can be transformed in a not-parametric index simply by changing the weights.

$I(P)$ satisfies the properties C, B, I and PM if and only if I_{row} in turn satisfies them. The new index will be then based on a "row-by-row" interpretation of mobility (note that the function $I_{row}(\cdot)$ is actually defined on the subset in \mathbb{R}^k of vectors describing discrete probability distributions: the *standard* $(k-1)$ -simplex Δ^{k-1} containing vectors with non-negative, summing-to-1 elements).

3 Mobility with entry/exit events

3.1 Reassessing TMs

Following Adelman (1958) and Duncan and Lin (1972), as a first step we propose to rearrange the TM for better visualizing movements in and out the sample. With closed data sets and k states, TMs classically have the form $P = \{p_{ij}\}_{i,j=1,\dots,k}$, where p_{ij} is defined as in Eq. 1. In the open case, we then consider k regular states together with the additional "outer" state O . Ingoing individuals come from and outgoing individuals go to the state O . The corresponding augmented TM has $k+1$ rows and columns as it follows:

$$P_{\beta,\delta} = \left[\begin{array}{ccc|c} p_{11}^* & \cdots & p_{1k}^* & \delta_1 \\ \vdots & \ddots & \vdots & \vdots \\ p_{k1}^* & \cdots & p_{kk}^* & \delta_k \\ \hline \beta_1 & \cdots & \beta_k & w \end{array} \right] = \left[\begin{array}{c|c} P^* & \delta^t \\ \hline \beta & w \end{array} \right] \quad (3)$$

where p_{ij}^* is the conditional transition probability between two regular states i and j , β is the vector containing the probabilities β_j that a new individual will appear in the j -state at time $t+1$, and δ is the vector of the probabilities δ_i that an individual will disappear from the i -th state. Lastly, w is the probability that a not-observed individual at time t does not enter in any regular state between t and $t+1$. Having observed only two consecutive steps, and with no additional theoretical assumptions, p_{ij}^* is evaluated as in Eq. 1 considering $k+1$ states, instead of k . Analogously, we set

$$\delta_i = \frac{\text{nr. of individuals exiting from } i}{\text{nr. of individuals starting from } i}.$$

Note that $p_{ij} = p_{ij}^*$ for every $j = 1, \dots, k$ if and only if $\delta_i = 0$. If the total number of individuals in and out the sample is known, β_j is also evaluated as a conditional percentage:

$$\beta_j = \frac{\text{nr. of individuals entering in } j}{\text{nr. of individuals being outside at time } t},$$

and $w = 1 - \sum_{j=1}^k \beta_j$. An important remark is now needed: in many cases the number of individuals waiting outside is not known (see Adelman, 1958; Duncan and Lin, 1972), and β_j is not evaluable as a conditioned percentage as δ_i . Estimating the birth probability when the number of individuals waiting outside is not observable could be matter for further research.

We stress again the fact that any theoretical model explaining $P_{\beta,\delta}$ is assumed. The augmented TM proposed in Eq. 3 contains probabilities calculated as observed conditional frequencies and it is relevant because it represents the usual and simplest way to elaborate data and measure mobility in empirical applications (see among others Jafry and Schuermann, 2004; Macchiarelli and Ward-Warmedinger, 2014). On this basis, the seemingly simplest way to measure mobility in presence of entry-exit phenomena consists in applying the chosen index directly on the $(k+1) \times (k+1)$ TM $P_{\beta,\delta}$. To avoid drawbacks caused by the matrix dimension⁴, mobility $P_{\beta,\delta}$ should be compared with the "closed" matrix given by

$$P_{00} = \left[\begin{array}{c|c} P & \underline{0}^t \\ \hline \underline{0} & 1 \end{array} \right].$$

Looking at the specific kind of mobility measured by a given index, we can guess the effect of the additional state on the mobility value. For example, the trace index provides a measure of the turbulence in the sample under study, which tends to increase with the number of communicant states. Then $I^{tr}(P_{\beta,\delta})$ is expected to be higher than $I^{tr}(P_{00})$.

In addition to the dependence from the number of states, the "raw" application of some mobility index on $P_{\beta,\delta}$ is affected by some pitfalls as listed in the following.

1. the potential increase/decrease of mobility depends only on the presence of an additional state, but not on the particular role such state plays in the whole dynamics. That is, the outer state is considered as a regular one, and birth/death events are treated as generic transitions among states.
2. There is no clear information about the direction, since the outer state generally is not defined to be worse or better than the others. In conse-

⁴ Dependence on the dimensions is a typical problem of descriptive statistics, as it happens for example with the R^2 in the linear regression framework and with the Cramer index measuring the degree of association of two variables.

quence of that, directional indices such as I^u , I^d and I^{dir} could be not applicable.

Instead of applying mobility indices directly on the augmented TM, we then propose to maintain the number k of regular states and to consider a new index $I_{\leftrightarrow}(\cdot)$ defined on $\mathbb{R}^{k \times k}$ such that β and δ appear as parameters to include In/Outgoings mobility.

3.2 Basic hypotheses for the new index

In the previous sections we illustrate the empirical ways to reorganize data and measure mobility when theoretical models are not assumed. We now suppose to have at disposal the values of Incumbents transition probabilities and of birth and death probability in any state. The new index we are going to propose does not depend from the way we have obtained such values and it can be applied both when probabilities are calculated as observed frequencies and when they are estimated through some theoretical model (for example, a Poisson distribution ruling the number of births in every state). Nevertheless, we are still not assuming any model. The only assumptions we need regard the relationship among individuals: we suppose that statistical units are independent one from each other and that individuals starting from a given state are homogeneous (i.e. mobility is state-by-state decomposable as in Sect. 2.2).

From now onward we will use the following notation: 1) $b_j(t)$ is the number of arrivals in the j -th state, that is the number of individuals missing at time t and recorded in j at time $t + 1$; 2) $d_i(t)$ is the number of departures from the i -th state, that is individuals recorded in i at time t and missing at time $t + 1$; 3) $\beta_j(t)$ and $\delta_i(t)$ are the corresponding probabilities, as before. When unnecessary, the time index t will be avoided for shortness.

We propose here some additional hypothesis about Incomings and Outgoings. From now on, we suppose to have at disposal the Incumbents transition probabilities p_{ij} and the birth/death probabilities for every state, not regarding at the way they have been calculated.

H1: For every state i the number of arrivals $b_i(t)$ at time t does not depend on the number of departures $d_i(t)$ at the same time from the same state, and vice versa (note that $b_i(t)$ could depend both on $b_i(t - 1)$ and $d_i(t - 1)$, but the analysis of such relationship goes beyond the scope of this work).

H2: Movements among the regular states at time t depend on the contemporaneous number of departures $d_i(t)$, and the outgoing phenomena represents an instantaneous loss in the capacity to evolve. Formally speaking, the transition probabilities p_{ij} derived from closed panels are actually conditional probabilities to move towards j , given the starting state i and given that moving individuals have survived from t to t . Then we can say that:

$$p_{ij}^* = (1 - \delta_i) \cdot p_{ij}, \forall i, j = 1, \dots, k. \quad (4)$$

We can see that mortality is an endogenous event conditioning the amount of movements among the regular states. In particular, it is proved that Outgoings make the mobility to decrease.

H3: The number of arrivals $b_i(t)$ at time t does not modify the Incumbents transition probabilities between t and t . It instead will disturb future movements because Incomings make change the number of individuals starting from every state at time $t + 1$. This fact can be proved using the well-known formula for calculating the distribution of individuals among the states. In fact, let $p_i(t)$ be the probability/frequency of individuals to in the i -th regular state at time t , and $p_o(t)$ the probability to be outside at the same time: it is well known that

$$p_j(t+1) = \sum_{i=1}^k p_i(t) \cdot p_{ij}^* + p_o(t) \cdot \beta_j(t), \forall j = 1, \dots, k. \quad (5)$$

The Incomings impact on the whole mobility is less neat than Outgoings. Nevertheless, we note that $d_j(t+1)$ increases linearly with β_j . Analogously to the Outgoings case, we propose to consider newborn individuals as a gain in the whole capacity to move. More details will be provided in the following sections.

Such hypotheses, joined with the results displayed in Sect. 2, form a procedure for constructing a new index able to measure mobility of both Incumbents and In/Outgoings. We remind that we aim to measure the mobility among the k regular states.

3.3 Outgoings mobility

According with H1, we can split and examine separately the ways to include Incomings or Outgoings in the whole mobility measure.

Firstly, we face the case in which only Outgoings and Incumbents are observed ($\beta = 0$). Outgoings mobility is in many sense more interesting to be treated than Incomings because in empirical unbalanced data sets they are more probable to be observed. In addition, according with H2, deaths between t and $t + 1$ correspond to a loss in the capacity to move among the k regular states, and consequently they modify the degree of mobility in the same span of time.

To obtain the mobility index with Outgoing I_δ we recall here the results shown in the previous sections. Being I_{row} defined on Δ^{k-1} , I_{row} should be not well-defined if applied on $(p_{i1}^*, \dots, p_{ik}^*)$, whose sum is equal to $1 - \delta_i \neq 1$. Reminding Eq. 4, we also observe the the most part of the functions I_{row} associated to the existing indices are linear with respect to the terms p_{ij} , and consequently $I_{row}(P_i^*) = (1 - \delta_i) \cdot I_{row}(P_i)$. We then suggest to include Outgoings by rescaling the mobility of Incumbents. Formally, we propose the index

$$I_\delta(P(t)) = C \cdot \sum_{i=1}^k (1 - \delta_i(t)) \cdot I_{row}(P_i(t)), \quad (6)$$

where the functional form of $I_{row}(\cdot)$ is given by the formula of the index we have chosen for the Incumbents group. The time index t is included to stress the fact that all the terms in Eq. 6 possibly depend on time. I_δ is coherent with H2. Furthermore, an explanation is provided for the choice to include the term $I_{row}(P_i)$ instead of $I_{row}(P_i^*)$, which could be not well-defined unless we consider O as a regular state.

3.4 Incomings mobility

We now consider the case in which only Incomings and Incumbents are observed ($\delta = 0$ and $p_{ij}^* = p_{ij}$). According with H3, we suppose that individuals entering between t and $t + 1$ do not intervene on the Incumbents mobility, but they make change the distribution among states at time $t + 1$.

Having in mind the decomposition of mobility measures as in Eq. 2, we suggest to include information about Incomings in the mobility index using the quantities $p_i(t)$ as weights to measure the mobility during the interval $[t, t + 1]$. Consequently, the mobility index with Incomings is

$$I_\beta(P(t)) = C \sum_{i=1}^k p_i(t) \cdot I_{row}(P_i(t)), \quad (7)$$

In Ferretti and Ganugi (2013) it is already highlighted the relevance of choosing the starting distribution for weighing the contribution to mobility of individuals moving according to P_t . Roughly speaking, since the whole mobility is a sort of weighted mean of contributes due to individuals starting from the k state, if a given state is empty at time t its contribute at that time should be null. In this case the role of $\omega_i = p_i(t)$ is twofold: on one hand more/less crowded states have a higher/lower role in the whole mobility; on the other hand Incomings contribute to the whole mobility according with their birth state. The index I_β is coherent with H3 (note that t is the starting time for evaluating the TM, and $p_i(t)$ includes Incomings born between $t - 1$ and t).

4 Properties of the new index

The new index we propose for measuring mobility in the case of open panels is then:

$$I_{\leftrightarrow}(P) = C \cdot \sum_{i=1}^k \omega_i \cdot I_{row}(P_i), \quad (8)$$

in which every term is possibly function of t and $\omega_i = p_i \cdot (1 - \delta_i)$ is the weight to be assigned to $I_{row}(P_i)$, as explained before.

Properties of I_{\leftrightarrow} derive from the properties of the function $I_{row}(\cdot)$, in particular:

- C) continuity of I_{row} implies continuity of I_{\leftrightarrow} ;
- B) boundedness of I_{row} implies boundedness of I_{\leftrightarrow} ;
- I) immobility of I_{row} implies immobility of I_{\leftrightarrow} (note that I_{row} satisfies immobility if $I_{row}(e_i) = 0$ where e_i is the i -th standard basis vector in \mathbb{R}^{k5});
- PM) perfect mobility of I_{row} implies perfect mobility of I_{\leftrightarrow} (note that a given TM P has the maximum mobility if and only if $I_{row}(P_t)$ is maximal for every $i = 1, \dots, k$).

⁵ In addition, we note that $I_{\leftrightarrow} = 0$ also if $p_i(t) = 0$ or $\delta_i = 1$, for every i . This is a degenerate, not-relevant case such that there are some empty states and individuals in the remaining states die immediately

An important remark regards the weights ω_i , since it is easy to verify that they do not sum to 1. Obviously, it is always possible to fix the sum using the normalized weights $\tilde{\omega}_i = \frac{\omega_i}{\sum \omega_i}$, but it has no significant impact on the properties of the new index. On the other hand we note that every ω_i has a specific role: in fact $\delta_i(t)$ is the probability to exit conditioned to the starting state i at time t , and $\sum_i p_i(t) \cdot (1 - \delta_i(t))$ is the global probability to be still alive at time $t + 1$ obtained through the Bayes formula. In consequence of that, Outgoings are considered as "null-mobility" individuals, and the whole value of the index decreases if the probability/percentage of deaths increases.

Lastly, let P_{β^p, δ^p} and Q_{β^q, δ^q} be matrices to be compared as before, equipped with the corresponding birth/ death probabilities, and let P and Q be the TMs evaluated on the closed subset of Incumbents. Choosing a classical index I (and the associated I_{row}) from the previous list, we suppose to find $I(P) < I(Q)$. It is important to remark that it is not necessarily true that $I_{\leftrightarrow}(P_{\beta^p, \delta^p}) < I_{\leftrightarrow}(Q_{\beta^q, \delta^q})$, except for the case $\beta^p = \beta^q$ and $\delta^p = \delta^q$, since I and I_{\leftrightarrow} are both weighted means of k terms $I_{row}(P_i)$ and $I_{row}(Q_i)$, but they are evaluated with different sets of weights. In this sense, we remark that I_{\leftrightarrow} exploits results about existing and well-known indices for measuring the Incumbents mobility, but it is a full-fledged mobility index revealing a specific order among TMs according with the related force of natality/mortality.

5 Case study: Labour Market mobility

As an empirical illustration we apply the new mobility index using EURO-STAT data about Italian and UK labor market's transition from 2010:q2 to 2016:q1. The data set contains the observed transition among three states: Unemployment (U), Employment (E) and Not-Active (N) (see Tab. 2 and 4). We define as "regular" any worker belonging to the Labour Force, that is moving between E and U , whereas the outer state O coincides with N . Consequently, birth and death events correspond to movements in and out the Labour Force. It is worth noting that such panels are open in an extended manner: individuals do not miss in any point of time (and n_{OO} is known), but they possibly move from/to the regular states to/from the outer state.

In literature, some seminal papers use a two-state representation ($E - U$) for analyzing the labour market transition (Gomes, 2012; Elsby *et al.*, 2013). In more recent papers instead the relevance of including transitions towards and from the Not-active state is stressed (Elsby *et al.*, 2015). Along the same

line, we propose here to compare the mobility measured on both the closed and open panels, with the specific aim to check the properties of the modified index proposed before.

Figure 1 displays the observed conditional percentages of transitions between Unemployment and Employment, both when Not-active workers are included (dotted lines) and non included in the sample under study (solid lines). On one hand, we can see that the presence of Not-active workers has a huge effects on the evaluation of transitions from Unemployment to Employment ($U \rightarrow E$). On the other hand, transitions from E to U ($E \rightarrow U$) are nearly indistinguishable both with two- and three-state representation of the labor market, particularly for UK (right side of Fig. 1).

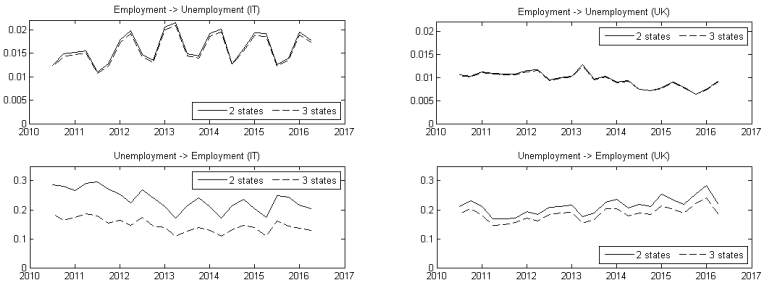


Figure 1: Transitions between Employment and Unemployment evaluated for 2-states (p_{EU} and p_{UE}) and 3-states (p_{EU}^* and p_{UE}^*) labor market representation).

Such difference is mainly due to the huge amount of transitions from U to N of Italian workers with respect to British workers. From Fig. 2 (lower side) we can see that the frequency of Italian workers moving to the Not-active state, starting from U , is on average equal to 37.6%, and around three times higher than British workers (12.8%). In regards of Incomings, it is noticeable that Italian workers transitions from N to U tends to increase with the time (Fig. 2, upper left panel). Lastly, it is worth to be remarked the scale difference between Incomings and Outgoings. For example, the percentage of Italian Outgoings from U are on average 7.5 times bigger than percentages of Incomings towards the same state.

In labour market analysis observed transition percentages are often used

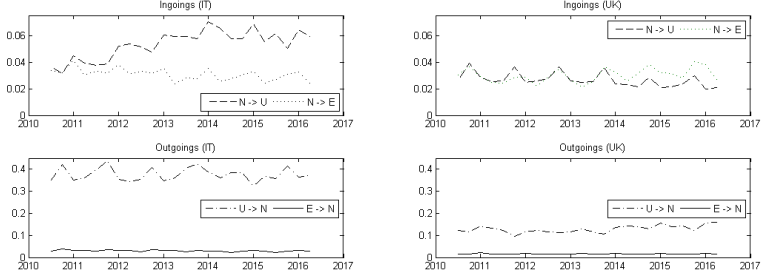


Figure 2: Transitions in and out from the Labor Force (β_E , β_U , δ_E and δ_U).

to have a (raw) estimate of flow rates of movements among the labour market states (see Gomes, 2012). Flow rates represent a measure of the labour market mobility, because they describe the "force" with whom workers can leave the current state (in particular Unemployment) and move towards an other state (preferably Employment). Then, we can use results shown in Fig. 2 to have a first look in the Italian and British labour market mobility. Italian workers are seemingly more mobile, also if unfortunately mobility seems to interest mainly unemployed and and inactive workers.

As an example, we choose to measure the Labour Market turbulence and the prevailing direction, using the functional form of I_{row} proposed for the above-mentioned trace index and directional index. We then use the following formulas:

$$I_{\leftrightarrow}^r(P(t)) = \sum_{i=E,U} p_i^*(t) \cdot (1 - \delta_i(t)) \cdot (1 - p_{ii}(t)) \text{ and}$$

$$I_{\leftrightarrow}^{dir}(P(t)) = \sum_{i=E,U} p_i^*(t) \cdot (1 - \delta_i(t)) \cdot \sum_{j=E,U} p_{ij}(t) \cdot \text{sign}(j - i) \cdot |j - i|,$$

where t indexes quarters, in comparison with the indices evaluated only on the active workers:

$$I^r(P(t)) = \sum_{i=E,U} p_i(t) \cdot (1 - p_{ii}(t)) \text{ and}$$

$$I^{dir}(P(t)) = \sum_{i=E,U} p_i(t) \cdot \sum_{j=E,U} p_{ij} \cdot \text{sign}(j - i) \cdot |j - i|.$$

In the formulas above, $p_i(t)$ and $p_i^*(t)$ represents the observed percentage of individuals in $i = E, U$ at the t -th quarter calculated respectively on the Labour Force and on the whole group of workers (see Tab. 3 and 5). Note that I^r differs from its classical definition for including the amount of individuals starting from every state as weights. In all the proposed indices, we provisionally discard the normalizing constant, because the definition of a suitably normalized index will be matter for further research. Despite they are not-normalized, the chosen indices are still comparable since they are defined on the same number of states.

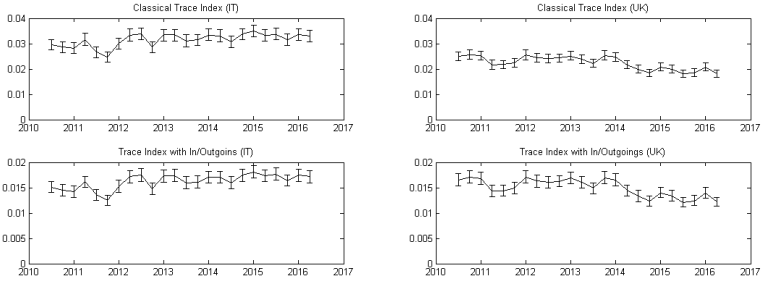


Figure 3: Trace Index measured with closed (upper side) and open (lower side) panels, for Italy (left) and UK (right).

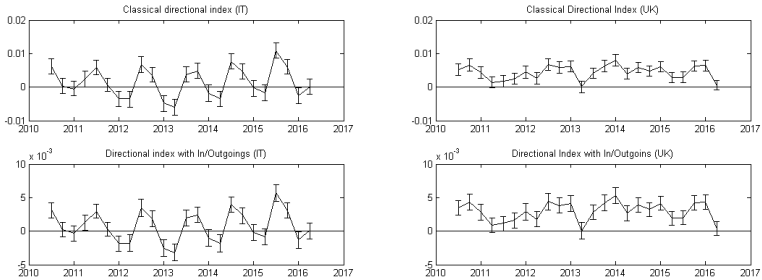


Figure 4: Directional Index measured with closed (upper side) and open (lower side) panels, for Italy (left) and UK (right).

Figg. 3 and 4 display the mobility values for both the closed and open dataset. Vertical lines represent the $IC_{97.5\%}$ obtained bootstrapping 999 fictitious samples. I and I_{\leftrightarrow} values seemingly differ only by a multiplicative constant. Actually this is not true, as revealed by Tab. 1 which contains the mobility values only for the second quarter in every year. For example, we can see that in 2010:q2 the mobility restricted to the Labour Force is higher in Italy than UK both measured with the trace and the directional index, whereas we find the opposite result when we measure mobility including not-active workers (that is, using a more exhaustive representation of the Labour Market). More generally, the UK trace index on the closed Labour Force is systematically lower than Italy, whereas turbulence measured with the open dataset shows comparable results in the whole considered temporal window. Besides, it is reasonable that indices measuring the same kind of mobility show the same trend in the time. In fact, mobility varies in the time not depending on the function which defines the index, and similar indices able to capture such variations are supposed to show a similar shape.

Table 1: Mobility indices evaluated for the second quarter of every year, from 2010 to 2015*.

Time	2010:q2			2011:q2			2012:q2		
	IT	UK	IT/UK	IT	UK	IT/UK	IT	UK	IT/UK
Labour Force									
I^r	0.0295	0.0250	118%	0.0265	0.0217	122%	0.0337	0.0242	139%
I^{dir}	0.0061	0.0053	116%	0.0058	0.0017	347%	0.0067	0.0067	100%
Active and not-active workers									
I^r_{\leftrightarrow}	0.0151	0.0166	91%	0.0135	0.0144	94%	0.0175	0.0161	109%
$I^{dir}_{\leftrightarrow}$	0.0031	0.0035	90%	0.0030	0.0011	267%	0.0035	0.0044	78%
Time	2013:q2			2014:q2			2015:q2		
	IT	UK	IT/UK	IT	UK	IT/UK	IT	UK	IT/UK
Labour Force									
I^r	0.0311	0.0223	140%	0.0308	0.0199	155%	0.0337	0.0181	187%
I^{dir}	0.0038	0.0042	91%	0.0077	0.0058	132%	0.0109	0.0030	370%
Active and not-active workers									
I^r_{\leftrightarrow}	0.0160	0.0149	107%	0.0160	0.0134	119%	0.0176	0.0121	145%
$I^{dir}_{\leftrightarrow}$	0.0020	0.0028	70%	0.0040	0.0039	102%	0.0057	0.0020	287%

*2016:q2 is not included in the EUROSTAT data downloaded on January 2017.

Comparing Italy and UK through I^r_{\leftrightarrow} and $I^{dir}_{\leftrightarrow}$, we see that the trace index occupies the same range for both the countries (from around 0.012 to around 0.018). Furthermore, Italy and UK show an opposite behaviour in terms of turbulence: Italian trace index increases in the time whereas British index decreases.

The main differences between the two countries are revealed when using the directional index. We remind that in the case of the directional mobility,

we focus on the prevailing direction in the dynamics under study, when states are ordered from the "worst" to the "best" one. The state-by-state contribute to mobility is measured through the function $I_{row}(P_i) = \sum_j p_{ij} \cdot \text{sign}(j - i) \cdot |j - i|$ and the whole index has positive/negative sign when the prevailing direction is towards a general improvement/worsening with respect to the current state. We see that the directional index is affected by a strong seasonality, especially in Italy. The main difference between Italy and UK regards the direction of workers mobility: British workers always evolve prevailing towards the Employment state (except for 2013:q1 and 2016:q1 in which mobility is null), whereas the Italian mobility has positive sign approximately only in the first six month of every considered year. On the other hand, we note that UK mobility seems to have a more static behaviour than Italy, for which we observe a seemingly increasing trend joined with the cyclical behaviour, starting from 2013. The inspection of such results from an economic and politic point of view could be matter for further research.

6 Conclusions

In this work we propose a new mobility index able to capture the effect of mobility of Incomings and Outgoings together with the Incumbents, that is of individual leaving and entering in the sample under study for birth/death phenomena. The index is obtained according with the following procedure: having defined k states on which transition matrices are built, we choose the kind of mobility we are interested in, for example turbulence or directional mobility, and we set a suitable function I_{row} able to measure the contribute to mobility of Incumbents starting from the i -th state, $i = 1, \dots, k$. The whole mobility value is a weighted sum of the k terms $I_{row}(P_i)$, where P_i is the i -th row of the transition matrix. Information about Incomings and Outgoings is included in the mobility measurement by setting suitable weights for the weighted sum. In particular, we assume (and formally prove) that Incomings and Outgoings make the mobility respectively to increase/decrease, and they are considered in the mobility index by choosing weights which are linearly increasing/decreasing with respect to the birth/death rate. Furthermore, we prove that Outgoings represents an instantaneous loss in the capacity to move, whereas Incomings have a delayed effect on mobility since they make change the number of individuals starting from every state at the next step.

The new index is applicable both when the transition matrix and the

birth/death probability coincide with observed frequencies and when they are estimated through some theoretical model.

The empirical example regards the Italian and UK Labour Market compared using an index of turbulence and an index based on the prevailing direction, applied on EUROSTAT data from 2010:q2 to 2016:q1. UK shows a more static behaviour of both the indices with respect of time and the mobility in the labour market has always a positive sign, which means that workers tends on average to move mainly towards the Employment state. Italy instead displays an increasing mobility but the direction in the Labour Market depends on seasons (towards Employment in Fall/Winter and towards Unemployment in Spring/Summer).

Further research will regard the problem of estimating the parameters needed for calculating the index, in particular the birth probability or birth rate. Possibly, we will model Incomings and Outgoings probability using a theoretical model, both in discrete and continuous time, and we will consequently adapt the index formula. Robustness of the index with respect to possible bias in the transition, birth and death probabilities could be also matter for further research.

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A EUROSTAT Labour market data -Italy-

Table 2: EUROSTAT Labor Market Transitions in Italy (thousand persons)

Time	2010:q2	2010:q3	2010:q4	2011:q1	2011:q2	2011:q3	2011:q4	2012:q1	2012:q2	2012:q3	2012:q4	2013:q1
IT_UU	1031	854	868	962	891	781	899	1208	1289	1194	1257	1561
IT_UE	413	333	316	395	374	291	304	347	476	378	336	323
IT_UN	780	860	638	772	841	832	658	820	964	1078	847	1061
IT_EU	271	323	328	338	239	277	387	430	319	293	450	467
IT_EE	21495	21402	21333	21463	21605	21584	21417	21402	21496	21543	21412	21276
IT_EN	606	878	734	721	641	796	717	688	559	812	693	722
IT_NU	747	645	934	806	773	804	1088	1092	1043	952	1238	1194
IT_NE	694	658	872	627	678	646	798	626	675	635	717	474
IT_NN	19130	19233	19186	19148	19214	19260	19018	18697	18538	18518	18508	18427
Time	2013:q2	2013:q3	2013:q4	2014:q1	2014:q2	2014:q3	2014:q4	2015:q1	2015:q2	2015:q3	2015:q4	2016:q1
IT_UU	1509	1324	1354	1699	1670	1462	1605	1790	1594	1369	1335	1527
IT_UE	409	420	365	349	455	452	410	376	530	443	366	392
IT_UN	1304	1285	1092	1164	1322	1188	961	1254	1178	1289	976	1134
IT_EU	319	310	411	431	274	343	417	415	271	303	425	389
IT_EE	21182	21162	21072	21111	21241	21255	21215	21234	21363	21509	21456	21446
IT_EN	572	691	659	623	456	652	693	650	461	619	693	645
IT_NU	1201	1177	1447	1317	1159	1170	1398	1098	1237	1004	1293	1171
IT_NE	573	560	728	511	553	618	673	484	539	621	657	484
IT_NN	18478	18643	18472	18424	18518	18482	18234	18270	18373	18348	18257	18226

Source: EUROSTAT database on Labour Market Transitions at http://appsso.eurostat.ec.europa.eu/nui/show.do?dataset=lfsl_jong_&lang=en

Table 3: Starting distributions Italy (percentages)

Time	2010:q2	2010:q3	2010:q4	2011:q1	2011:q2	2011:q3	2011:q4	2012:q1	2012:q2	2012:q3	2012:q4	2013:q1
Whole sample distribution												
P_U^*	4.92	4.53	4.03	4.71	4.65	4.21	4.11	5.24	6.02	5.84	5.37	6.47
P_E^*	49.53	50.02	49.54	49.79	49.68	50.05	49.73	49.70	49.33	49.88	49.62	49.37
P_N^*	45.54	45.45	46.43	45.50	45.66	45.75	46.16	45.06	44.66	44.28	45.02	44.16
Incumbents* distribution												
p_U	6.22	5.18	5.18	5.86	5.47	4.67	5.23	6.65	7.49	6.72	6.79	7.97
p_E	93.78	94.82	94.82	94.14	94.53	95.33	94.77	93.35	92.51	93.28	93.21	92.03
Time	2013:q2	2013:q3	2013:q4	2014:q1	2014:q2	2014:q3	2014:q4	2015:q1	2015:q2	2015:q3	2015:q4	2016:q1
Whole sample distribution												
P_U^*	7.07	6.65	6.16	7.04	7.55	6.80	6.53	7.50	7.25	6.81	5.89	6.72
P_E^*	48.46	48.63	48.56	48.58	48.13	48.77	48.95	48.93	48.51	49.29	49.66	49.50
P_N^*	44.46	44.72	45.28	44.38	44.32	44.43	44.52	43.56	44.24	43.89	44.45	43.78
Incumbents* distribution												
p_U	8.19	7.51	7.41	8.68	8.99	8.14	8.52	9.10	8.94	7.67	7.21	8.08
p_E	91.81	92.49	92.59	91.32	91.01	91.86	91.48	90.90	91.06	92.33	92.79	91.92

The number of Incumbents in every quarter is given by IT_UU+IT_EU+IT_UE+IT_EE in Tab. 2

B EUROSTAT Labour market data -United Kingdom-

Table 4: EUROSTAT Labor Market Transitions in United Kingdom (thousand persons)

Time	2010:q2	2010:q3	2010:q4	2011:q1	2011:q2	2011:q3	2011:q4	2012:q1	2012:q2	2012:q3	2012:q4	2013:q1
UK_UU	1717	1635	1695	1757	1780	1853	1925	1877	1813	1747	1788	1768
UK_UE	462	494	456	356	361	384	466	420	479	469	491	378
UK_UN	306	283	357	319	302	237	311	321	305	282	301	315
UK_EU	301	293	324	314	309	309	330	338	272	290	299	373
UK_EE	28020	28235	28321	28431	28449	28389	28336	28403	28464	28662	28889	28892
UK_EN	447	405	623	456	394	471	532	481	412	421	458	503
UK_NU	394	580	413	372	385	539	364	381	412	544	374	358
UK_NE	451	540	424	365	359	425	420	325	431	516	387	312
UK_NI	13948	13705	13669	14017	14128	13913	13871	14066	14128	13863	13827	13948
Time	2013:q2	2013:q3	2013:q4	2014:q1	2014:q2	2014:q3	2014:q4	2015:q1	2015:q2	2015:q3	2015:q4	2016:q1
UK_UU	1793	1693	1671	1551	1457	1372	1269	1192	1193	1169	1095	1053
UK_UE	415	497	516	405	407	367	430	362	335	396	435	300
UK_UN	291	257	340	322	306	260	312	252	254	218	280	259
UK_EU	283	299	264	278	224	216	233	272	241	195	230	283
UK_EE	28877	28937	29182	29433	29553	29767	29896	30007	30094	30178	30373	30581
UK_EN	422	427	529	459	444	439	572	517	496	476	575	503
UK_NU	371	535	343	341	317	423	304	318	348	446	287	310
UK_NE	373	540	473	383	461	567	470	463	421	604	558	395
UK_NN	14090	13803	13791	13978	14023	13890	13899	14041	14135	13976	13860	14033

Source: EUROSTAT database on Labour Market Transitions at <http://appsso.eurostat.ec.europa.eu/mui/show.do?dataset=lfsl-jong-&lang=en>

Table 5: Starting distributions United Kingdom (percentages)

Time	2010:q2	2010:q3	2010:q4	2011:q1	2011:q2	2011:q3	2011:q4	2012:q1	2012:q2	2012:q3	2012:q4	2013:q1
Whole sample distribution												
p_U^i	5.40	5.22	5.42	5.24	5.26	5.32	5.80	5.62	5.56	5.34	5.51	5.25
p_E^i	62.48	62.67	63.24	62.95	62.74	62.70	62.72	62.69	62.39	62.77	63.33	63.54
p_N^i	32.13	32.11	31.34	31.81	32.01	31.98	31.48	31.69	32.05	31.89	31.16	31.20
Incumbents* distribution												
p_U	7.14	6.94	6.98	6.85	6.93	7.23	7.70	7.40	7.39	7.11	7.24	6.83
p_E	92.86	93.06	93.02	93.15	93.07	92.77	92.30	92.60	92.61	92.89	92.76	93.17
Time	2013:q2	2013:q3	2013:q4	2014:q1	2014:q2	2014:q3	2014:q4	2015:q1	2015:q2	2015:q3	2015:q4	2016:q1
Whole sample distribution												
p_U^i	5.33	5.21	5.36	4.83	4.60	4.23	4.24	3.81	3.75	3.74	3.80	3.38
p_E^i	63.05	63.13	63.63	63.99	64.04	64.32	64.79	64.94	64.88	64.73	65.37	65.74
p_N^i	31.62	31.66	31.01	31.18	31.36	31.46	30.97	31.25	31.37	31.53	30.83	30.89
Incumbents* distribution												
p_U	7.04	6.97	6.91	6.18	5.89	5.48	5.34	4.88	4.80	4.90	4.76	4.20
p_E	92.96	93.03	93.09	93.82	94.11	94.52	94.66	95.12	95.20	95.10	95.24	95.80

The number of Incumbents in every quarter is given by UK_UU+UK_EU+UK_UE+UK_EE in Tab. 4

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